*Bulgarian Chemical Communications, Volume 48, Special Issue E (pp. 342 - 347) 2016*

# Soliton regime of propagation of optical pulses in isotropic medium under the influence of third order of linear dispersion and dispersion of nonlinearity

D. Y. Dakova[1, *], A. M. Dakova[1, 2], V. I. Slavchev[2, 3], L. M. Kovachev[2]

[1]*Faculty of Physics, University of Plovdiv "Paisii Hilendarski", 24 Tsar Asen Str., 4000 Plovdiv, Bulgaria*
[2]*Institute of Electronics, Bulgarian Academy of Sciences,72 Tzarigradsko shossee,1784 Sofia, Bulgaria*
[3]*Faculty of Pharmacy, Medical University - Plovdiv, Bul. Vasil Aprilov 15-A, 4002 Plovdiv, Bulgaria*

In the recent years the evolution of ultrashort broad-band optical pulses in nonlinear dispersive media attracts a considerable attention. For attosecond and phase-modulated femtosecond laser pulses the following condition is satisfied: $\Delta\omega\approx\omega_0$. One of the most commonly used equation in optics, to describe the propagation of optical pulses in planar and one-dimensional waveguides, is the nonlinear Schrodinger equation (NSE). It is derived for narrow-band pulses ($\Delta\omega<<\omega_0$) and works very well for nanosecond and picosecond laser pulses. As a result of different linear and nonlinear mechanisms, in femtosecond and attosecond region it is easy to obtain broad-band pulses where $\Delta\omega\approx\omega_0$. In this case, it is more convenient to use the general nonlinear amplitude equation (NAE) which works properly for narrow-band as well as broad-band light pulses. In the present work a theoretical model of the evolution of broad-band optical pulses in single-mode silica ($SiO_2$) fibers is presented. In the frames of ultrashort optics the influence of effects of dispersion and nonlinearity of the medium are significant. This requires the inclusion of additional terms in NAE that govern the third order of linear dispersion and dispersion of nonlinearity. In present paper we found a new exact analytical soliton solution of NAE. It is shown that it is possible to observe a soliton as a result of the dynamic balance between the effects of higher order of dispersion and nonlinearity of optical $SiO_2$ fibers. Obtained results are important for better understanding of the evolution of broad-band optical pulses, propagating in medium under the influence of third order of linear dispersion and dispersion of nonlinearity.

**Keywords:** $SiO_2$ single-mode fibers, third order of linear dispersion, the dispersion of nonlinearity, nonlinear amplitude equation, soliton solution, broad-band optical pulses

## INTRODUCTION

In the last two decades the evolution of femtosecond and attosecond optical pulses with broad-band spectrum in nonlinear dispersive medium is of a considerable interest for the scientific community [1-3]. Its study is a result of the growing needs of ultrafast high intensity optics. One of the most commonly used equations to describe the propagation of laser pulses in one-dimensional structures and planar waveguides is the nonlinear Schrodinger equation [4-9]. In the frames of ultrashort optics ($T_0<1ps$) it is usually modified by adding terms that govern the third order of the linear dispersion (TOD) and the dispersion of nonlinearity [4,10]:

$$i\frac{\partial A}{\partial \xi}+\frac{1}{2}\frac{\partial^2 A}{\partial \tau^2}+i\delta\frac{\partial^3 A}{\partial \tau^3}+k_0 n_2 L_D\left[|A|^2 A+is\frac{\partial}{\partial \tau}(|A|^2 A)\right]=0 \quad (1)$$

where $\delta=k'''/(6T_0|k''|)$, $s\approx1/(T_0\omega_0)$, $L_D=T_0^2/|k''|$.

It is well known that, for such pulses, it is necessary TOD ($k'''$) to be taken into account even when group velocity dispersion (GVD) is nonzero $k''\neq0$. As a result of that, the shape of the pulse becomes asymmetric with an oscillatory structure on one of its edges, depending on the sign of $k'''$ (Fig.1) [4].

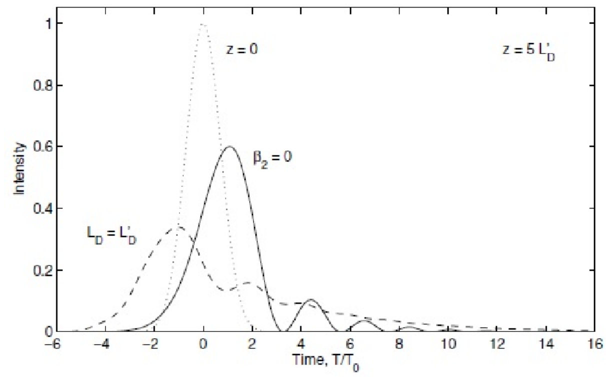


**Fig.1.** Gaussian pulse in the presence of TOD, $L'_D=T_0^3/|k'''|$, [4]

If pulses propagate at the zero-dispersion wavelength, the effects of TOD are dominant and lead to deep oscillations with intensity dropping to zero at the leading edge of the pulse when $k'''<0$ and at its trailing edge when $k'''>0$, respectively. In soliton regime of propagation, the main effect of TOD, on the evolution of laser pulses, is to shift theirs peak position linearly with distance $z$ [4]. When $k'''>0$ the soliton peak is slowed down and when $k'''<0$ it speeds up. The shift is considerable in attosecond and femtosecond region. Effects of TOD on the propagation of optical solitons are widely discussed in [11-16].

Self-steepening (dispersion of nonlinearity)

* To whom all correspondence should be sent:
ddakova2002@yahoo.com

($s=1/\omega_0 T_0$) is a higher-order nonlinear effect that results from the intensity dependence of group velocity and leads to an asymmetry in the shape and spectrum of ultrashort pulses. Their peaks shift toward the trailing edges, moving at lower speed than wings (Fig.2) [4].

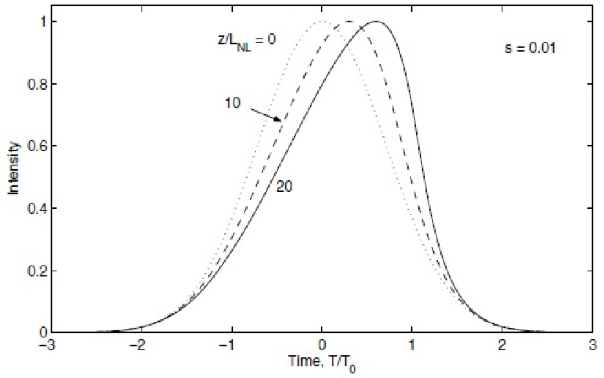


**Fig.2.** Dispersionless case of self-steepening of Gaussian pulse, $z=10L_{NL}$ and $20L_{NL}$, $s=1/\omega_0 T_0$ [4]

Self-steepening could create an optical shock wave, which formation is delayed in dispersionless case as a result of fiber losses. In soliton regime of propagation self-steepening leads to a spectral and temporal shift of pulses. These effects have been studied extensively in [17-21].

It is important to mention that the separate influence of third order of linear dispersion and dispersion of nonlinearity alter the pulses parameters and breakup higher-order solitons into their constituents [4]. Thus, it was interesting to ask the question: *Is it possible a soliton to be formed in such a medium as a result of the compensation of the simultaneous influence of these two effects*?

Our quick review shows that the dynamics of optical solitons in different nonlinear dispersive media is well studied, based on the nonlinear Schrodinger equation [10,22-26]. Authors in [10] investigated the standard and modified NSE numerically. It is shown that for broad-band femtosecond optical pulses, under certain conditions, it is possible a soliton to be observed. It maintains its shape even in the presence of self-steepening and third order of dispersion as a result of the balance between the higher-order nonlinear and dispersive effects (Fig.3) [10].

The NSE very well describes the propagation of slowly varying amplitude function of the envelope of narrow-band pulses in optical fibers but in the frames of broad-band optics (phase-modulated femtosecond and attosecond laser pulses) it is necessary to work with the more general nonlinear amplitude equation NAE [27,28] which differs from NSE (1) with two additional terms. The biggest advantage of NAE is that it can be applied in both cases – for pulses with broad-band and narrow-band spectrum.

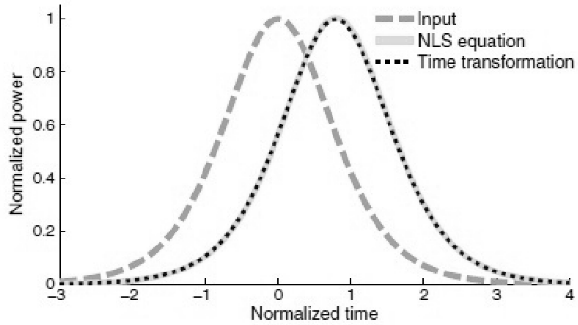


**Fig.3.** Intensity profile of *sech* input pulse (dashed line) $z=0$ and $z=10L_D$, $s=1/4\pi$, $T_0=10fs$, $\delta=0.02$. Predictions of modified NSE are shown by the solid curve [10]

In present paper we propose a theoretical model based on the evolution of light pulses with a broad-band spectrum in optical fibers under the effects of third order of linear dispersion and dispersion of nonlinearity. In our work losses and Raman scattering of the medium are neglected. New exact analytical soliton solution of NAE is found by using mathematical method described in [28].

## BASIC EQUATION

The 3D vector amplitude equation that describes the evolution of short optical pulses in Kerr-type nonlinear dispersive isotropic medium has the form [27]:

$$\begin{aligned}&\Delta\vec{A}+2ik_0\left[\frac{\partial\vec{A}}{\partial z}+\frac{1}{v_{gr}}\frac{\partial\vec{A}}{\partial t}+\left(\frac{2n_2k_0}{\omega_0}+k_0\frac{\partial n_2}{\partial\omega}\right)\frac{\partial}{\partial t}\left(\left|\vec{A}\right|^2\vec{A}\right)\right]+\\&\frac{2i}{v_{gr}}\hat{\Theta}\frac{\partial\vec{A}}{\partial t}+2k_0\hat{\Theta}\vec{A}-\frac{1}{v_{gr}^2}\frac{\partial^2\vec{A}}{\partial t^2}+2n_2k_0^2\left|\vec{A}\right|^2\vec{A}=0\end{aligned}\qquad(2)$$

where:

$$\hat{\Theta}=\sum_{n=2}^{\infty}\frac{1}{n!}k^{(n)}\left(i\frac{\partial}{\partial t}\right)^n=-\frac{1}{2}k''\frac{\partial^2}{\partial t^2}-i\frac{k'''}{3!}\frac{\partial^3}{\partial t^3}+\ldots\qquad(3)$$

$$\begin{aligned}&k_0=k(\omega),\ n=n(\omega_0),\ k'=\frac{\partial k}{\partial\omega},\\&k''=\frac{\partial^2 k}{\partial\omega^2},\ k'''=\frac{\partial^3 k}{\partial\omega^3},\ v_{gr}=\frac{1}{k'}\end{aligned}\qquad(4)$$

In the equation above (2) $\vec{A}$ is the vector amplitude function of the pulse envelope; $t$ is time; $\omega_0$, $k$, $v_{gr}$, $n$ and $n_2$ are the carrier frequency, wave number, group velocity, linear and nonlinear refractive index of the medium, respectively; $\Delta$ is the operator of Laplace.

- It is assumed that:

$$\vec{A} = A\vec{n} \quad , \qquad (5)$$

where $\vec{n}$ is the single constant vector and $A = \left|\vec{A}\right|$ is the magnitude of the amplitude function. Thus:

$$\Delta\vec{A} = \vec{n}\Delta A \qquad (6)$$

By the substitutions above equation (2) can be presented in scalar form.

- The evolution of one-dimensional pulses in $SiO_2$ single-mode fiber is examined. So, we are not interested in their transverse size. It is assumed that the axis *Oz* coincides with the axis of symmetry of the medium. Therefore, the magnitude of the amplitude function *A* is presented as a function of only two variables *t* and *z*, i.e.:

$$A = A(t, z) \qquad (7)$$

- We work in local time coordinate system:

$$T = t - z / v_{gr} \quad . \qquad (8)$$

By these assumptions equation (2) can be written in scalar form:

$$i\frac{\partial A'}{\partial \xi} + \frac{1}{2\alpha}\left[\frac{\partial^2 A'}{\partial \xi^2} - 2\frac{\partial^2 A'}{\partial \xi \partial \tau}\right] + \frac{|\beta_2|}{2\alpha}\frac{\partial^2 A'}{\partial \tau^2} + \frac{iC}{2\alpha}\frac{\partial^3 A'}{\partial \tau^3} + \gamma|A'|^2 A' + i\gamma s\frac{\partial}{\partial \tau}(|A'|^2 A') = 0, \qquad (9)$$

where

$$A = A'\sqrt{A_0}, \quad \xi = \frac{z}{z_0}, \quad \tau = \frac{T}{T_0}, \quad z_0 = v_{gr}T_0,$$

$$\alpha = k_0 z_0, \quad |\beta_2| = k_0|k''|v_{gr}^2, \quad |\beta_3| = \frac{k_0|k'''|v_{gr}^2}{3T_0},$$

$$\gamma = \alpha_0 n_2 |A_0|^2, \quad s = \left[\frac{2}{\omega_0 T_0} + \frac{1}{\chi^{(3)}T_0}\frac{\partial \chi^{(3)}}{\partial \omega}\right],$$

$$C = \frac{|\beta_2|}{\alpha}(1+\theta), \quad \theta = \frac{k_0 c}{n_0}\frac{|k'''|}{|k''|} = \frac{\alpha}{3}\frac{L_D}{L'_D}$$

Using the substitutions above the amplitude equation (2) is presented in dimensionless form. The parameters $z_0$ and $T_0$ represent the initial longitudinal length and pulse duration; $A_0$ is its initial amplitude. The constant $\alpha$ $(\alpha>1)$ characterizes the number of harmonic oscillations at level *1/e* from the maximum of the pulse amplitude. The coefficients $\beta_2$ and $\beta_3$ are connected with second and third order of the linear dispersion. It is assumed that higher orders of linear dispersion are negligible compared to the second and third ones, characterized by $k''$ and $k'''$. Therefore, we consider only the first two terms of the differential operator (3). The GVD of the medium is anomalous, i.e. $k''<0$ and $\beta_2<0$: $\beta_2= -|\beta_2|$. The parameter $\gamma$ depends on the nonlinear refractive index $n_2$.

Equation (9) is a nonlinear partial differential equation that describes the change in the magnitude of the amplitude function of pulses, propagating in $SiO_2$ single-mode fibers. It presents the effects of linear and nonlinear dispersion of the medium and describes the evolution of the amplitude function of the electric field even when the laser pulse admits few optical cycles inside. Here, it is important to mention that for nanosecond and picosecond light pulses, the coefficient $1/2\alpha$ in front of the brackets in equation (9) is quite small and it can be neglected. In this case, NAE tends to the modified NSE. That is the reason why, NSE describes very well the evolution of narrow-band laser pulses in single-mode fibers. Obviously, in femtosecond and attosecond regions, the coefficient $1/2\alpha$ is significant and the two additional terms - the second derivative ($\partial^2 A'/\partial\xi^2$) and the mixed derivative ($\partial^2 A'/\partial\xi\partial\tau$), must be taken into account [29,30].

## SOLITON SOLUTION OF THE NONLINEAR AMPLITUDE EQUATION

We search for a solution of the scalar nonlinear amplitude equation (9) of the form:

$$A'(\xi,\tau) = \Phi(x)\exp(ia\tau + ib\xi), \quad x = \tau + u\xi \quad , \qquad (10)$$

where *a*, *b* and *u* are constants which are due to be defined, *Φ(x)* is a real function. It is important to mention that constant *u* has a meaning of velocity which shifts the peak of pulses.

As a first step, the expression (10) is substituted in equation (9). A complex nonlinear ordinary differential equation of third order and third degree with respect to the unknown function *Φ(x)* is obtained. In the next step, the real and imaginary parts on both sides of the equation are equalized. Thus, the following two differential equations are obtained:

$$Re: \ \Phi''\left[u^2 - 2u + |\beta_2| - 3aC\right] - \Phi\left[2\alpha b + b^2 - 2ab + a^2|\beta_2| - Ca^3\right] + \Phi^3 2\alpha\gamma[1 - as] = 0 \qquad (11)$$

$$Im: \ \Phi'''C + 3\Phi^2\Phi' 2\alpha\gamma s - \Phi'\left[2b + 2au + 3Ca^2 - 2a|\beta_2| - 2bu - 2\alpha u\right] = 0 \qquad (12)$$

The coefficients in front of the corresponding derivatives and degrees of the function $\Phi$ in equations (11) and (12) are dimensionless. Equation (12) is integrated with respect to the variable $x$ to lower its order. Thus, it takes the following form:

$$\Phi'' C - \Phi\left[2b + 2au + 3Ca^2 - 2a|\beta_2| - 2bu - 2\alpha u\right] + \Phi^3 2\alpha\gamma s = B = const \qquad (13)$$

As it can be noticed, equations (11) and (13) are of the same type and are referred to the same unknown function. In that sense, they should match. Moreover, the coefficients in front of the corresponding derivatives and degrees of $\Phi$ must be the same. Thus, the integration constant $B$ is zero [28,31]. By equalizing these coefficients constants $a$, $b$ and $u$ are obtained:

$$a = \frac{\alpha}{2} - 1,$$
$$b_{1,2} = -\left(1 + \frac{\alpha}{2}\right) \mp \sqrt{1 - |\beta_2|\left[1 - (1+\theta)\left(\frac{3}{2} - \frac{2}{\alpha}\right)\right]} \pm \frac{\alpha}{2}\sqrt{1 - |\beta_2|\left(1 - \left(\frac{1+\theta}{2}\right)\right)}, \qquad (14)$$
$$u = 1 \pm \sqrt{1 - |\beta_2|\left[1 - (1+\theta)\left(\frac{3}{2} - \frac{2}{\alpha}\right)\right]}$$

Once the three constants are defined in a way that equations (11) and (13) match, for the unknown real function $\Phi=\Phi(x)$ the following ordinary nonlinear differential equation of second order is obtained:

$$\Phi'' - \eta^2\Phi + 2N^2\Phi^3 = 0 \quad , \qquad (15)$$

where

$$\eta^2 = \frac{3\alpha^2}{4} - 1 - \frac{\alpha^2}{|\beta_2|(1+\theta)}\left[1 + |\beta_2| \pm \sqrt{1 - 2|\beta_2|\left[1 - (1+\theta)\left(1 - \frac{1}{\alpha}\right)\right] + |\beta_2|^2\left[1 - 2(1+\theta)\left(1 - \frac{1}{\alpha}\right) + (1+\theta)^2\left(\frac{3}{4} - \frac{1}{\alpha}\right)\right]}\right] \qquad (16)$$

The number of the soliton is given as follow:

$$N^2 = \frac{\alpha s}{C}\gamma \qquad (17)$$

The soliton solution of equation (15) has a well-known form [4-7]:

$$\Phi = \eta\sqrt{\frac{C}{\alpha\gamma s}}\,\mathrm{sec}h(\eta x) \qquad (18)$$

The constant $\eta$ has a meaning of amplitude. The solution (18) has a physical meaning when $\eta$ is real and $\eta > 0$. This condition can be satisfied by the appropriate selection of the parameters of optical pulses and $SiO_2$ fibers.

Having in mind the expressions (14) and substituting (18) in (10), a new exact analytical soliton solution of NAE (9), including the effects of self-steepening and TOD, is obtained:

$$A'(\xi,\tau) = \eta\sqrt{\frac{C}{\alpha\gamma s}}\,\mathrm{sech}[\eta(\tau + u\xi)]\,\mathrm{e}^{\iota[a\tau + b\xi]} \qquad (19)$$

This result differs from the standard soliton solution of NSE. The additional phase term found in solution (19) leads to a significant temporal shift of the soliton peak position. Constants (14) depend on the number of optical cycles inside the pulse and the parameters of the medium. The biggest advantage of the soliton solution (19) is that, it can be used for more accurate description of the propagation of narrow-band as well as broad-band optical pulses in isotropic one-dimensional nonlinear dispersive media.

## NUMERICAL CALCULATIONS

For laser pulse with $\lambda=1{,}55\mu m$ propagating in fused silica ($n_0 \approx 1{,}47$; $|k''| \approx 1{,}8.10^{-26}\,s^2/m$; $|k'''| \approx 10^{-40}\,s^3/m$) [4] it is defined that $k_0=4{,}05.10^6\ m^{-1}$, $|\beta_2| \approx 3.10^{-3}$ and $\theta \approx 1{,}53$. In that case, it is assumed that $\partial\chi^{(3)}/\partial\omega \approx 0$.

Thus, the following approximate solution of equation (9) is obtained:

$$\Phi \approx \eta\,\mathrm{sec}h(\eta x), \quad \eta \approx \alpha, \quad u \approx |\beta_2|\left(\frac{5{,}2}{\alpha} - 3{,}4\right)$$
$$a = \frac{\alpha}{2} - 1, \quad b \approx -\left(1 + \frac{\alpha}{2}\right) + |\beta_2|\left(3{,}9 - \frac{7{,}8}{\alpha}\right), \quad s \approx 2/\alpha \qquad (20)$$

On Fig.4 and Fig.5 numerical calculations of expression (20) are shown. It is observed a soliton with amplitude proportional to the parameter $\alpha$, characterizing the number of harmonic oscillations under the pulse envelope. In Fig.4 the amplitude of initial *sech* pulse with time duration $T_0=10fs$ at $z=0$ is presented.

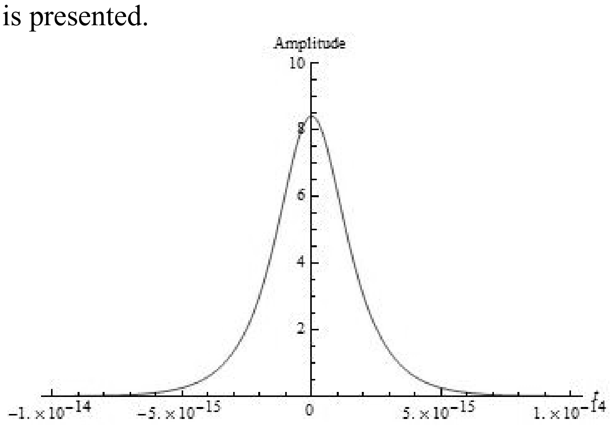


**Fig.4.** Amplitude of the *sech* pulse with $T_0=10fs$ and $z=0$

In Fig.5 is shown the same femtosecond *sech* pulse at distance $z=z_0$.

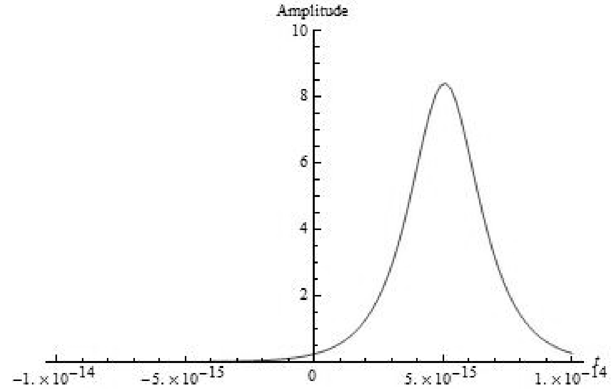


**Fig.5.** Amplitude of the *sech* pulse with $T_0=10fs$ and $z=z_0$

The shift in the temporal position of the soliton peak can be clearly seen. Nevertheless, the pulse keeps its shape as a result of the dynamic balance between the effects of third order of linear dispersion and dispersion of nonlinearity. As it was mentioned before, this shift is significant in attosecond and femtosecond regions [4,10]. For laser pulses with many oscillations under the envelope, two additional terms in equation (9) do not impact the soliton evolution and do not lead to a noticeable temporal shift. Decreasing the number of oscillations, the effect becomes important. This phenomena is explained in the frames of the applied mathematical model [29,32] and the influence of higher-order nonlinear and dispersive effects [4,10].

## CONCLUSION

In the present paper, the evolution of optical pulses with broad-band spectrum in nonlinear dispersive $SiO_2$ medium is reviewed. A new exact analytical soliton solution of NAE (9) is found, in which the effects up to third order of linear dispersion and dispersion of nonlinearity are included. The expression (19) differs significantly from the standard soliton solution of NSE – the constants $\eta$ and $u$, connected with the amplitude and the velocity of the temporal shift, respectively, depend on the coefficients, characterizing the second and third order of the linear dispersion and the nonlinearity of the medium, as well as the number of harmonic oscillations at level $1/e$ of maximum of the amplitude.

The analytical solution (19) and the numerical calculations of expression (20) answer to our question: *Is it possible a soliton in single-mode fibers to be formed under the simultaneous influence of TOD and self-steepening effects*? Fig.4 and Fig.5 show that in such media it is possible a soliton to be observed. The pulse is stable and keeps its shape as a result of the dynamic balance between the higher-order nonlinear and dispersive effects.

## ACKNOWLEDGMENTS

The present work is supported by project NI15-FFIT005, Faculty of Physics, Plovdiv University "Paisii Hilendarski".

## REFERENCES

1 P. Gibbon, Short Pulse Laser Interactions with Matter, London: Imperial College Press, (2007).
2 P. Tchofo Dinda, A. B. Moubissi, and K. Nakkeeran, *Phys. Rev. E* 64, 016608, (2001).
3 J. Santhanam and G. P. Agrawal, *IEEE J. Sel. Topics Quantum Electron.* 8, 632, (2002).
4 G. P. Agrawal, Nonlinear fiber optics, Academic Press, INC, New York, (2007).
5 Y. S. Kivshar and G. P. Agrawal, Optical Solitons: From Fibers to Photonic Crystals, San Diego: Academic Press, (2003).
6 F. Abdullaev, S. A. Darmanian, P. Khabibullaev, Optical solitons, London: Springer-Verlag, Berlin, (1993).
7 A. Hasegawa, F. Tappert, Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion*, Appl. Phys. Lett.*, 23, 142, (1973).
8 J. S. Aitchison et al., *J. Opt. Soc. Am. B*, 8 1290, (1991).
9 A. Barthelemy, S. Maneuf and C. Froehly, *Opt. Commun*, 55 201, (1985).
10 Y. Xiao, D. N. Maywar, G. P. Agrawal, New approach to pulse propagation in nonlinear dispersive optical media, *JOSA B*, Vol. 29, Issue 10, pp. 2958-2963, (2012).
11 P. K. Wai, C. R. Menyuk, H. H. Chen, and Y. C. Lee, *Opt. Lett.* 12, 628 (1987).
12 M. Desaix, D. Anderson, and M. Lisak, *Opt. Lett.* 15, 18, (1990).
13 V. K. Mezentsev and S. K. Turitsyn, *Sov. Lightwave Commun.* 1, 263, (1991).
14 Y. S. Kivshar, *Phys. Rev. A* 43, 1677, (1981); Opt. Lett. 16, 892, (1991).
15 V. I. Karpman, Phys. Rev. E 47, 2073, (1993); *Phys. Lett. A* 181, 211, (1993).
16 A. Peleg and Y. Chung, J. Phys. A 26, 10029, (2003).
17 N. Tzoar and M. Jain, *Phys. Rev.* A 23, 1266, (1981).
18 E. A. Golovchenko, E. M. Dianov, A. M. Prokhorov, and V. N. Serkin, *JETP Lett.* 42, 87, (1985).

19 K. Ohkuma, Y. H. Ichikawa, and Y. Abe, *Opt. Lett.* 12, 516, (1987).
20 A. M. Kamchatnov, S. A. Darmanyan, and F. Lederer, *Phys. Lett. A* 245, 259, (1998).
21 W. P. Zhong and H. J. Luo, *Chinese Phys. Lett.* 17, 577, (2000).
22 M. Mirzazadeh et al., *Journal of Nonlinear Optical Physics and Materials*, 24 1 1550006, (2015).
23 Q. Zhou et al., *Laser Physics*, 25 2 025402, (2015).
24 A. Sardar et al., *Journal of Nanoelectronics and Optoelectronics*, 11 3 382-387, (2016).
25 H. M. Li, J. Q. Zhao and L. Y. You, Indian Journal of Physics, 89 10 1065-1075, (2015).
26 Vivek K. Sharma, *Indian Journal of Physics*, 1-6, (2016).
27 R. W. Boyd, Nonlinear optics, Third Edition, Academic Press, Orlando, (2003).
28 A. M. Dakova, D. Y. Dakova, Evolution of optical pulses with broadband spectrum in a nonlinear dispersive media, *Bulg. J. Phys.* 40, 182–185, PACS codes: 42.81.-I, (2013).
29 A. M. Dakova, D. Y. Dakova, L. M. Kovachev and V. I. Slavchev, Comparison between the solutions of the general nonlinear amplitude equation and the modified Schrödinger equation, *Journal of Russian Laser Research*, Volume 37, Number 2, March (2016), p. 103-111.
30 D. Dakova, A. Dakova, V. Slavchev, L. Kovachev, Solitons in non-paraxial optics, *Journal of Optoelectronics and Advanced Materials*, Vol. 18, Iss. 5-6, May – June (2016), p. 435-439.
31 S. Krasteva and D. Dakova, *University of Plovdiv „Paisii Hilendarski" Scientific studies Physics*, 37 4, (2012).
32 A. Dakova, D. Dakova, L. Kovachev, Comparison of soliton solutions of the nonlinear Schrödinger equation and the nonlinear amplitude equation, *Proc. SPIE 9447, 18th International School on Quantum Electronics: Laser Physics and Applications*, 94471A (January 8, 2015); doi:10.1117/12.2177906, (2015).